MINISTRY OF EDUCATION AND SCIENCE OF UKRAINE
NATIONAL TECHNICAL UNIVERSITY
«DNIPRO POLYTECHNIC»

PROBLEMS OF STATICS

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PROCEDURE FOR SOLUTION OF PROBLEMS OF STATICS

1. Choose the body whose equilibrium should be examined. For the problem to lend itself to solution, the given and required forces, or their equivalents, should all be applied to the body whose equilibrium is being examined (for instance, if the problem is to determine a load acting on a support, we can examine the equilibrium of the body experiencing the reaction of the support, which is equal in magnitude to the required load).

If the given forces act on one body and the required on another, it may be necessary to examine the equilibrium of each body separately, or even of some intermediary bodies as well.

2. Isolate the body from its constraints and draw the given forces and the reactions of the removed constraints. Such a drawing is called a free-body diagram (FBD) and is drawn separately.

3. State the conditions of equi1ibrium. The statement of these conditions depends on the force system acting on the free body and the method of solution (graphical or analytical).

4. Determine the unknown quantities, verify the answer and analyze the results. In solving a problem, it is important to have a carefully drawn diagram, which helps to choose the correct method of solution and prevents errors in stating the conditions of equilibrium.

P r o b l e m. A crane held in position by a journal bearing A and a thrust bearing B carries a load P. Neglecting the weight of the structure determine the reactions R_A and R_B of the constraint if the jib is of length L and $AB = h$.

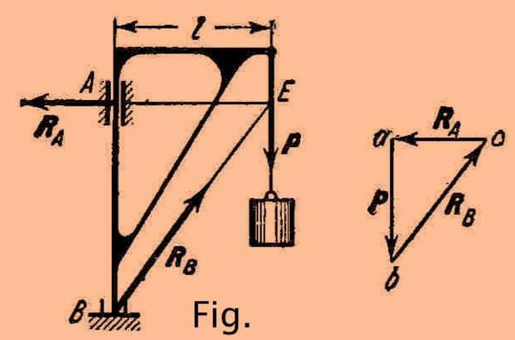


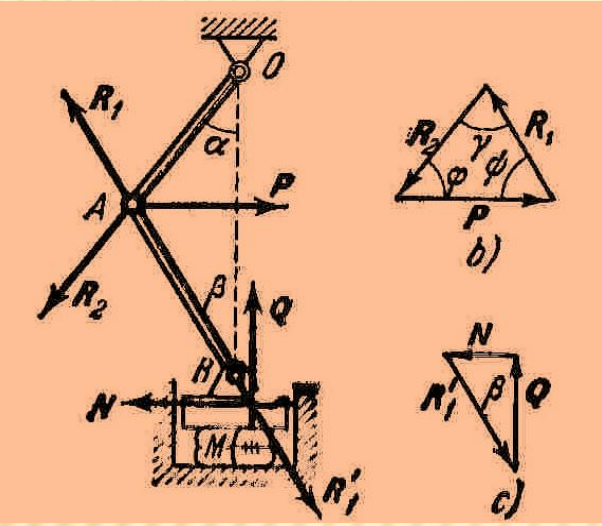
Fig.

S o l u t i o n. To solve the problem by the graphical method draw a close triangle *abc* with forces $\boldsymbol{P}$, $\boldsymbol{R_A}$, $\boldsymbol{R_B}$ as its sides starting with the given force $\boldsymbol{P}$. From the similarity of triangles *abc* and *ABE* we obtain:

$$\frac{R_A}{P} = \frac{1}{h}, \frac{R_B}{P} = \frac{\sqrt{h^2 + l^2}}{h}, \quad \text{whence} \quad R_A = \frac{1}{h}P, R_B = \sqrt{1 + \frac{l^2}{h^2}P}.$$

The loads acting on the journal bearing A and the thrust bearing B are respectively equal in magnitude to R_A and R_B but opposite in sense. The greater the ratio $l{:}h$ the greater the load acting on the constraints.

P r o b l e m. A horizontal force ***P*** is applied to hinge *A* of the toggle-press. Neglecting the weight of the rods and piston, determine the force exerted by the piston on body *M* when the given angles are α and β.



S o l u t i o n. First consider the equilibrium of the hinge *A* to which the given force ***P*** is applied. Regarding the hinge as a free body, we find that also acting on it are the reactions $\boldsymbol{R_1}$ and $\boldsymbol{R_2}$ of the rods directed along them. Construct a force triangle (Fig.b). Its angles are $\varphi=90^\circ-\alpha$, $\psi=90^\circ-\beta$, $\gamma=\alpha+\beta$. By the law of sines we have:

$$\frac{R_1}{\sin\varphi}=\frac{P}{\sin\gamma}, R_1=\frac{P\cos\alpha}{\sin(\alpha+\beta)},$$

Now consider the equilibrium of the piston, regarding it as a free body. Acting on it are three forces: $\boldsymbol{R'_1}=-\boldsymbol{R_1}$ exerted by rod *AB*, the reaction ***N*** of the wall, and the reaction ***Q*** of the pressed body. The three forces are in equilibrium, consequently they are concurrent. Constructing a triangle with the forces as its sides, (Fig.c), we find: $Q=R'_1\cos\beta$.

P r o b l e m. The bracket *ABCD* is in equilibrium under the action of two parallel forces ***P*** and ***P'*** making a couple. Determine the load on the supports if *AB* = *a* = 15 cm, *BC* = *b* = 30 cm, *CD* = *c* = 20 cm, and *P* =*P'* = 300 N.

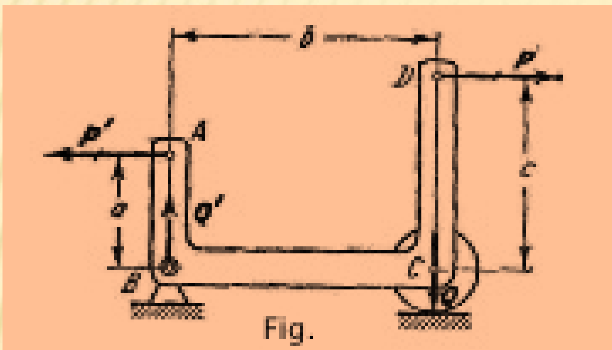


Fig.

S o l u t i o n. Replace couple (***P***,***P'***) with an equivalent couple (***Q***,***Q'***) whose two forces are directed along the reactions of the constraints. The moments of the two couples are equal, i.e., *P(c-a)=Qb*, consequently the loads on the constraints are:

$$Q = Q' = \frac{c-a}{b}P = 50N$$

and are directed as shown in the diagram.

P r o b l e m. A couple of moment m_1 acts on gear *1* of radius r_1 in Fig.a. Determine the moment m_2 of the couple which should be applied to gear *2* of radius r_2 in order to keep the system in equilibrium.

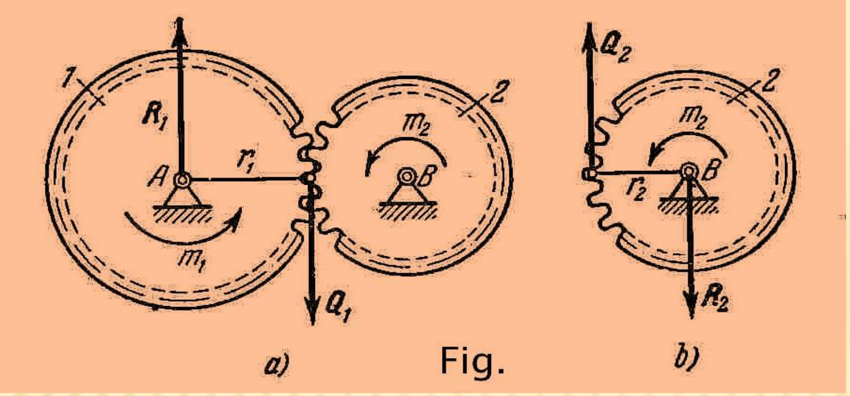


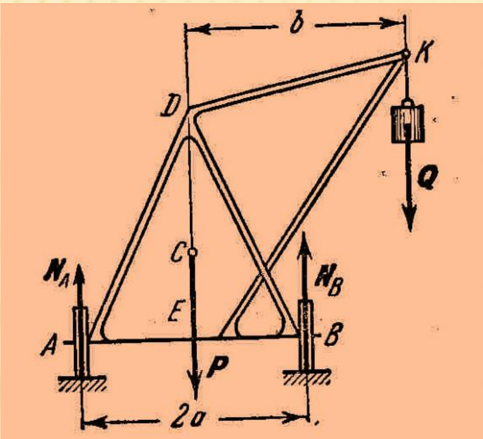
Fig.

S o l u t i o n. Consider first the conditions for the equilibrium of gear *1*. Acting on it is the couple of moment m_1 which can be balanced only by the action of another couple, in this case the couple ($\boldsymbol{Q_1}$, $\boldsymbol{R_1}$) created by the force $\boldsymbol{Q_1}$ exerted on the tooth of gear *1* by gear *2*, and the reaction $\boldsymbol{R_1}$ of the axle *A*.

So we have $m_1+(-Q_1r_1)=0$, or $Q_1=m_1/r_1$.

Consider now the condition for the equilibrium of gear *2*. We know that gear *1* acts on gear *2* with a force $\boldsymbol{Q_2}= -\boldsymbol{Q_1}$ (Fig.b), which together with the reaction of axle *B* makes a couple ($\boldsymbol{Q_2}$,$\boldsymbol{R_2}$) of moment $-Q_2r_2$. This couple must be balanced by a couple of moment m_2 acting on gear *2*; from equation of equilibrium we have: $m_2+(-Q_2r_2)=0$. Hence, as $Q_2=Q_1$, or $m_2 = \frac{r_2}{r_1} m_1$.

P r o b l e m. The travelling crane weighs $P = 4t$, its center of gravity lies on DE, it lifts a load of weight $Q = 1t$, the length of the jib (the distance of the load from DE) is $b = 3.5$ m, and the distance between the wheels is $AB = 2a = 2.5$ m. Determine the force with which the wheels A and B act on the rails.



S o l u t i o n. Consider the equilibrium of the crane-and-load system taken as a free body: the active forces are $\boldsymbol{P}$ and $\boldsymbol{Q}$, the unknown forces are the reactions $\boldsymbol{N_A}$ and $\boldsymbol{N_B}$ of the removed constraints.

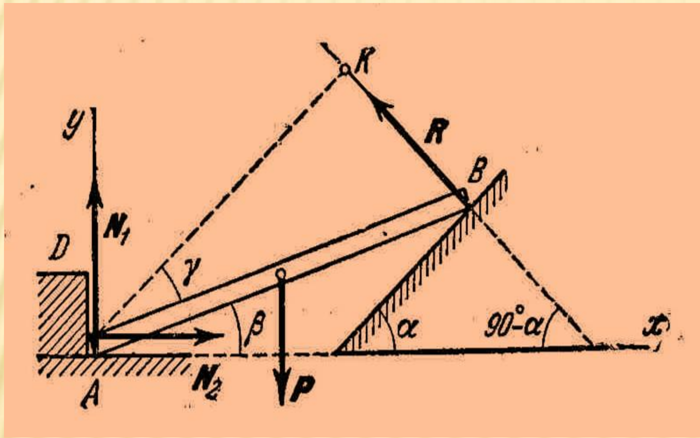
Taking A as the center of the moments of all the forces and projecting the parallel forces on a vertical axis, we obtain

$$-Pa + N_B \cdot 2a - Q(a+b)0,\ N_A + N_B - P - Q = 0,$$

whence $$N_A = \frac{P}{2} - \frac{Q}{2}\left(\frac{b}{a} - 1\right) - 1.1\ t; \quad N_B = \frac{P}{2} + \frac{Q}{2}\left(\frac{b}{a} + 1\right) = 3.9\ t.$$

From the solution we see that at $Q = \frac{a}{b-a}P = 2.22\ t$ the reaction $\boldsymbol{N_A}$ is zero and the left wheel no longer pressed on the rail. If the load Q is further increased the crane will topple over.

P r o b l e m. One end of a uniform beam *AB* weighing *P N* rests at *A* against a corner formed by a smooth horizontal surface and block *D*, and at *B* on a smooth plane inclined α degree to the horizontal. The beam's inclination to the horizontal is equal to β. Determine the pressure of the beam on its three constraints.

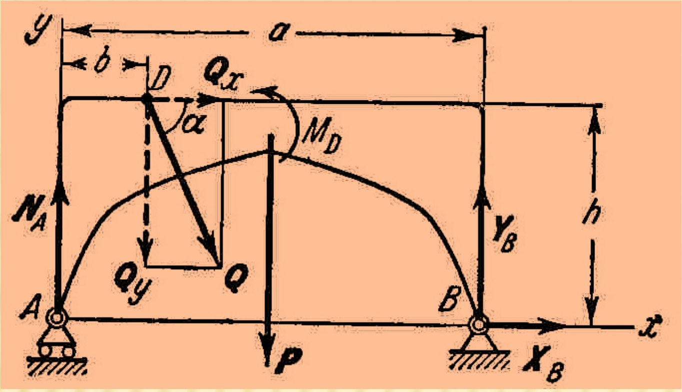


S o l u t i o n. Consider the equilibrium of the beam as a free body. Acting on it are the given force $\boldsymbol{P}$ and the reactions $\boldsymbol{R}$, $\boldsymbol{N_1}$, and $\boldsymbol{N_2}$ of the constraints directed normal to the respective surfaces. Draw the coordinate axes and write the equilibrium equations, taking the moments about *A*.

The equilibrium equations: $N_2 - R\sin\alpha = 0, N_1 - P + R\cos\alpha = 0,$ $-Pa\cos\beta + 2Ra\cos\gamma = 0$, $(\gamma = \alpha - \beta)$. Whence

$$R = \frac{P\cos\beta}{2\cos\gamma} = \frac{P\cos\beta}{2\cos(\alpha-\beta)}, N_1 = P\left[1 - \frac{\cos\alpha\cos\beta}{2\cos(\alpha-\beta)}\right], N_2 = P\frac{\sin\alpha\cos\beta}{2\cos(\alpha-\beta)}.$$

P r o b l e m. Acting on a symmetrical arch of weight $P = 8\ T$ is a set of forces reduced to a force $Q = 4\ T$ applied at D and a couple of moment $M_D = 12\ T\text{-}m$. The dimensions of the arch are $a = 10\ m$, $b = 2\ m$, $h = 3\ m$, and $\alpha = 60°$. Determine the reactions of the pin B and the roller A.



S o l u t i o n. In this problem it is more convenient to use eqs., taking the moments about A and B and the force projections on axis Ax, and each equation will contain one unknown force.

Writing the equilibrium equations and taking into account that $|Q_x| = Q\cos\alpha$, and $|Q_y| = Q\sin\alpha$, we obtain:

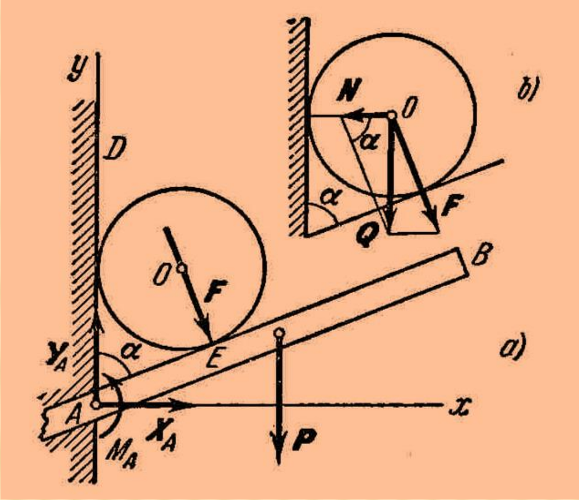
$$X_B + Q\cos\alpha = 0, \quad Y_B a - P\frac{a}{2} - hQ\cos\alpha - bQ\sin\alpha + M_D = 0,$$

$$-N_A a + P\frac{a}{2} - hQ\cos\alpha + (a-b)Q\sin\alpha + M_D = 0.$$ Whence we find

$$X_B = -Q\cos\alpha = -2\ T, \quad Y_B = \frac{P}{2} + Q\frac{b\sin\alpha + h\cos\alpha}{a} - \frac{M_D}{a} \approx 4.09\ T,$$

$$N_A = \frac{P}{2} + Q\frac{(a-b)\sin\alpha - h\cos\alpha}{a} + \frac{M_D}{a} \approx 7.37\ T.$$

P r o b l e m. The beam in Fig. is embedded in a wall at an angle $\alpha = 60°$ to it. The length of the portion *AB* is *b = 0.8* m and its weight is *P = 1000 N*. The beam supports a cylinder of weight *Q = 1800 N*. The distance *AE* along the beam from the wall to the point of contact with the cylinder is *a = 0.3* m. Determine the reactions of the embedding.



S o l u t i o n. Consider the equilibrium of the beam as a free body. Acting on it are force $\boldsymbol{P}$ applied halfway between *A* and *B*, force $\boldsymbol{F}$ applied perpendicular to the beam at *E* (but not force $\boldsymbol{Q}$, which is applied to the cylinder, no to the beam!), and the reactions of the embedding, indicated by the rectangular components $\boldsymbol{X_A}$ and $\boldsymbol{Y_A}$ and a couple of moment $\boldsymbol{M_A}$.

From the Fig. b we obtain: $F = \frac{Q}{\sin\alpha}$.

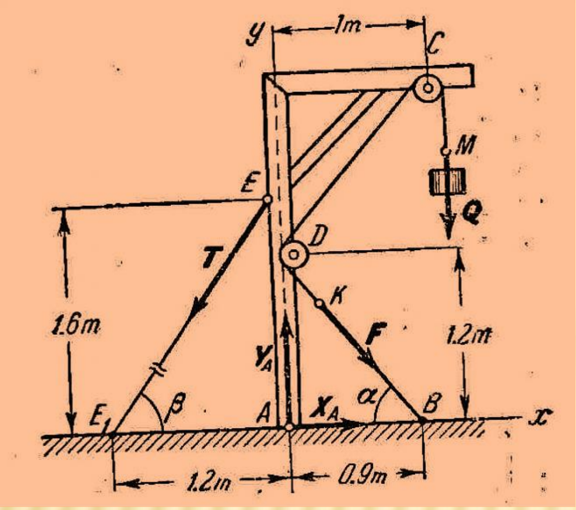
Writing the eqs. of equilibrium and substituting the value of *F*, we have:

$$X_A + Q\cot\alpha = 0, \qquad Y_A - Q - P = 0, \qquad M_A - Q\frac{a}{\sin\alpha} - P\frac{b}{a}\sin\alpha = 0.$$

Whence $X_A = -Q\cot\alpha = -1038\,N,$ $\qquad Y_A = P + Q = 2800\,N,$

$$M_A = Q\frac{a}{\sin\alpha} + P\frac{b}{2}\sin\alpha = 969\,N - m.$$

P r o b l e m. A string supporting a weight $Q = 2400\ N$ passes over two pulleys C and D as shown in Fig. The other end of the string is secured at B, and the frame is kept in equilibrium by a guy wire EE_1. Neglecting the weight of the frame and friction in the pulleys, determine the tension in the guy wire and the reactions at A, if the constraint at A is a smooth pivot allowing the frame to turn about its axis. The dimensions are as shown in the diagram.



S o l u t i o n. Consider the whole system of the frame and the portion $KDCM$ of the string as a single free rigid body. Acting on it are: the weight $\boldsymbol{Q}$, the tension $\boldsymbol{F}$ in section DB of the string, the reactions $\boldsymbol{T}$, $\boldsymbol{X_A}$, and $\boldsymbol{Y_A}$ of the constraints. The internal forces cancel each other. As the friction of the pulleys is neglected $F = Q$.

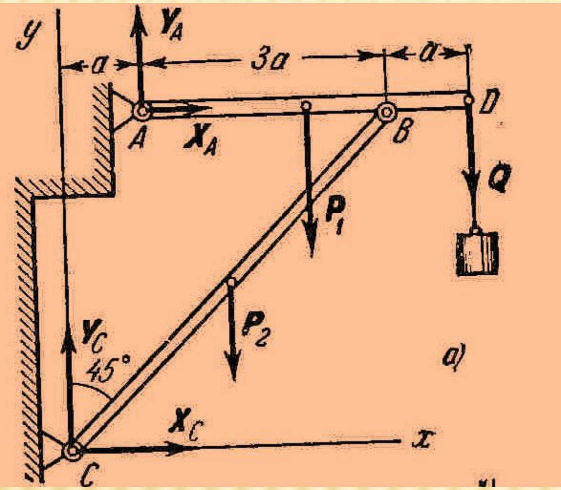
From the triangles AEE_1 and ADB we get: $EE_1 = 2.0$ m, $DB = 1.5$ m, whence $\sin\alpha = \sin\beta = 0.8$, $\cos\alpha = \cos\beta = 0.6$, and $\alpha = \beta$.

Equations of equilibrium:

$$0.6Q - 0.6T + X_A = 0, -Q - 0.8Q - 0.8T + Y_A = 0, -1.0Q - 0.72Q + 0.96T = 0,$$

whence $T = \dfrac{43}{24}Q = 4300\ N,\qquad X_A = \dfrac{19}{40}Q = 1140\ N,\qquad Y_A = \dfrac{97}{30}Q = 7760\ N.$

P r o b l e m. The horizontal member *AD* of the bracket weighs P_1 = 150 N, and the inclined member *CB* weights P_2 = 120 N. Suspended from the horizontal member at *D* is a load of weight *Q* = 300 N. Both members are attached to the wall and to each other by smooth pins (the dimensions are shown in the diagram). Determine the reactions at *A* and *C*.



S o l u t i o n. Considering the bracket as a whole as a free body, we find the acting on it are the given forces $\boldsymbol{P_1}$, $\boldsymbol{P_2}$, $\boldsymbol{Q}$ and the reactions of the supports $\boldsymbol{X_A}$,$\boldsymbol{Y_A}$, $\boldsymbol{X_C}$, $\boldsymbol{Y_C}$. But with its constraints removed the bracket is no longer rigid body, because the members can turn about pin *B*. On the other hand, by the principle of solidification, if it is in equilibrium the forces acting on it must satisfy the conditions of static equilibrium.

We may therefore write the corresponding equations:

$$\sum F_{kx} = X_A + X_C = 0, \qquad \sum F_{ky} = Y_A + Y_C - P_1 - P_2 - Q = 0,$$

$$\sum m_A(F_k) = X_C 4a - Y_C a - P_2 a - P_1 2a - Q4a = 0\,.$$

We find that the three equations contain four unknown quantities $\boldsymbol{X_A}$,$\boldsymbol{Y_A}$, $\boldsymbol{X_C}$, $\boldsymbol{Y_C}$. Let us therefore investigate additionally the equilibrium conditions of member *AD* (Fig.b). Acting on it are forces $\boldsymbol{P_1}$ and $\boldsymbol{Q}$ and the reactions $\boldsymbol{X_A}$, $\boldsymbol{Y_A}$, $\boldsymbol{X_B}$, and $\boldsymbol{Y_B}$. If we write the required fourth equation for the moments of these forces about *B* we shall avoid two more unknown quantities, $\boldsymbol{X_B}$ and $\boldsymbol{Y_B}$. We have

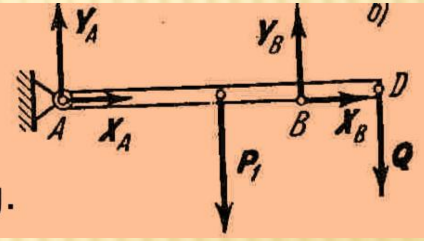


Fig.

$$\sum m_B(F_k) = -Y_A 3a + P_1 a - Qa = 0\,.$$

Solving the system of four equations (starting with the last one) we find:

$$Y_A = \frac{1}{3}(P_1 - Q) = -50\ N,\ Y_C = \frac{2}{3}P_1 + P_2 + \frac{4}{3}Q = 620\ N,$$

$$X_C = \frac{2}{3}P_1 + \frac{1}{2}P_2 + \frac{4}{3}Q = 560\ N, X_A = -X_C = -560\ N.$$

P r o b l e m. An evenly distributed force of intensity q_0 N/m acts on a cantilever beam whose dimensions are shown in Fig. Neglecting the weight of beam and assuming the forces acting on the embedded portion to be distributed according to a linear law, determine the magnitude of the maximum intensities q_m and q'_m of the given forces if $b=2a$.

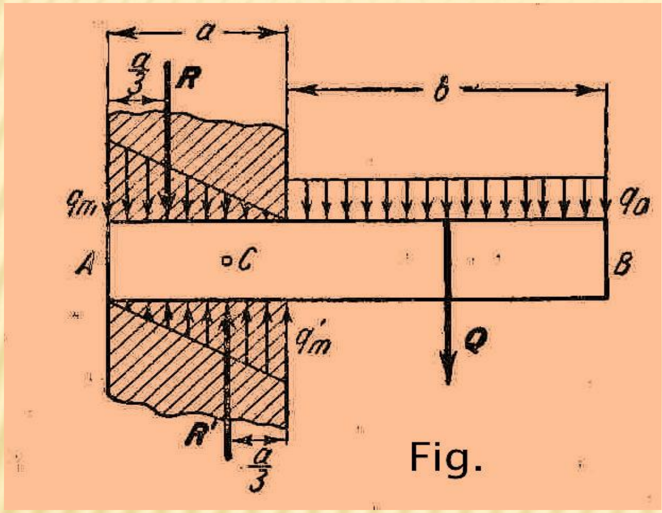


Fig.

S o l u t i o n. Replace the distributed forces by their resultants $\boldsymbol{Q}$, $\boldsymbol{R}$, and R'.

$$Q = q_0 b\,,\ R = \frac{1}{2} q_m a\,,\ R' = \frac{1}{2} q'_m a.$$

Now we can write the equilibrium conditions for the parallel forces acting on the beam:

$$\sum F_{ky} = Q + R - R' = 0, \qquad \sum m_C(F_k) = R\frac{a}{3} - Q\left(\frac{b}{2} + \frac{a}{3}\right) = 0.$$

Substituting the values of Q, R, and R' and solving the equations, we obtain:

$$q_m = \left(3\frac{b^2}{a^2} + 2\frac{b}{a}\right) q_0; \quad q'_m = \left(3\frac{b^2}{a^2} + 4\frac{b}{a}\right) q_0.$$

P r o b l e m. A dam of rectangular cross section of height h and width a rests on a horizontal foundation AB (Fig.). The weight of a unit volume of the impounded water is γ, and the weight of a unit volume of the dam filling is γ_1. Assuming the forces acting on the foundation to be linearly distributed (according to a trapezoidal law), determine the minimum and maximum intensities (q_1 and q_2) of the forces.

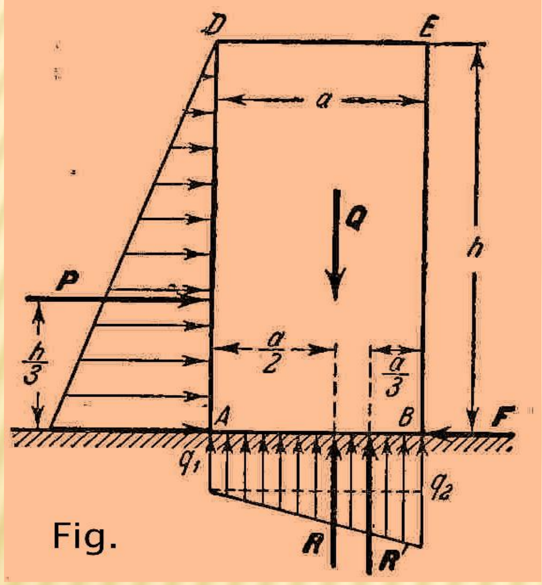


Fig.

S o l u t i o n. Consider a section of the dam of unit length perpendicular to the plane of the diagram. Consider further the forces acting on the section as if they were applied at the median cross section shown in Fig. These forces are: the pressure of the water $\boldsymbol{P}$, the weight of the dam $\boldsymbol{Q}$, the vertical reaction of the foundation, which we shall denote by two forces $\boldsymbol{R}$ and $\boldsymbol{R'}$, and the horizontal reaction $\boldsymbol{F}$.

The maximum intensity of the water loading is $q_m = \gamma h$, and $P = 1/2 q_m h = 1/2\, \gamma h^2$.

The component $\boldsymbol{R}$ of the reaction is $R = q_1 a$. Component $\boldsymbol{R'}$ is the resultant of linearly distributed forces with intensity varying from zero to (q_2-q_1); hence $R' = (q_2 - q_1)\frac{a}{2}$. The weight of a unit length of the dam is $Q = \gamma_1 a h$, whence

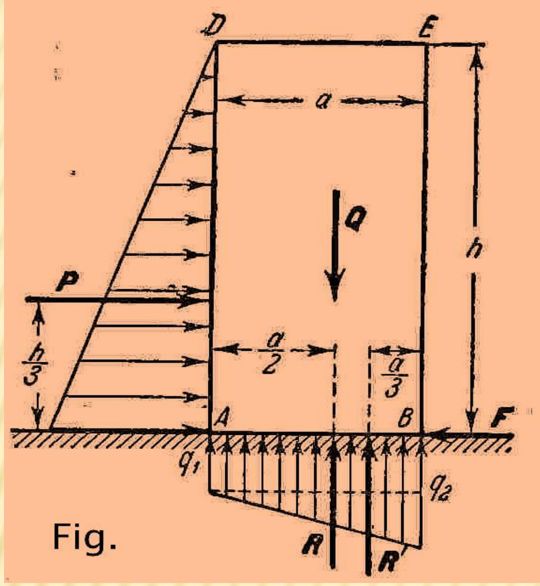


Fig.

$$P = \frac{1}{2}\gamma h^2, \qquad Q = \gamma_1 a h, \qquad R = q_1 a,$$

$$R' = (q_2 - q_1)\frac{a}{2}.$$

Writing now the equilibrium equations and taking the moments about the point of application of force ***R***, we obtain:

$$\sum F_{kx} = P - F = 0, \qquad \sum F_{ky} = R + R' - Q = 0,$$

$$\sum m_O(\boldsymbol{F_k}) = -P\frac{h}{3} + R'\frac{a}{6} = 0.$$

The first equation gives the horizontal reaction ***F***. From the second and third equations we find:

$$R' = 2P\frac{h}{a}; \quad R + R' = Q.$$

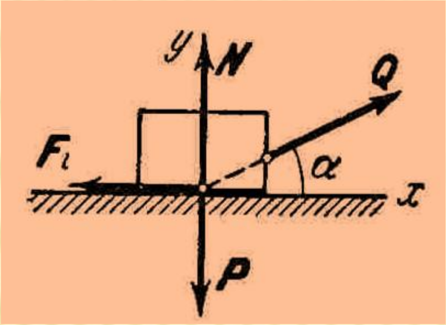
Substituting the values of the forces in the equations we obtain a simultaneous system of two equations:

$$q_2 - q_1 = 2\gamma\frac{h^3}{a^2}, \qquad q_2 + q_1 = 2\gamma_1 h,$$

whence

$$q_1 = \left(\gamma_1 - \frac{h^2}{a^2}\gamma\right)h, \qquad q_2 = \left(\gamma_1 + \frac{h^2}{a^2}\gamma\right)h.$$

P r o b l e m. A load of weight P = 100 N rests on a horizontal surface. Determine, the force Q that should be applied at an angle $\alpha = 30°$ to the horizontal to move the load from its place, if the coefficient of static friction for the surfaces of contact is $f_0 = 0.6$.



S o l u t i o n. According to the conditions of the problem we have to consider the position of impending motion of the load. In this position acting on it are forces $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{N}$ and $\boldsymbol{F_l}$. Writing the equilibrium equations in terms of the projections on the coordinate axes, we obtain:

$$Q\cos\alpha - F_l = 0; \quad N + Q\sin\alpha - P = 0.$$

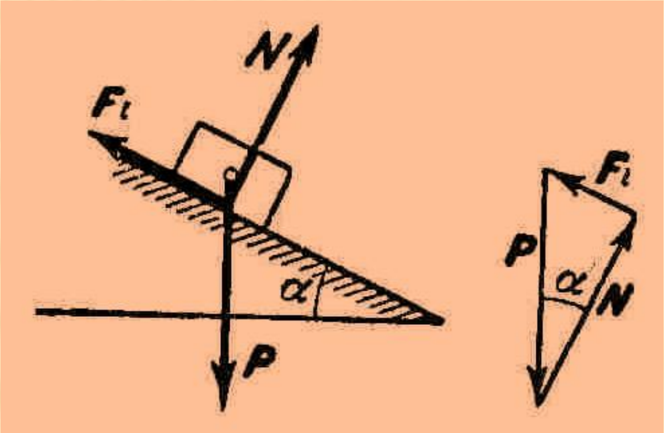
From the second equation $N = P - Q\sin\alpha$, whence:

$$F_l = f_0 N = f_0(P - Q\sin\alpha).$$

Substituting this value of $\boldsymbol{F_l}$ in the first equation, we obtain finally:

$$Q = \frac{f_0 P}{\cos\alpha + \sin\alpha} \approx 52\ N.$$

P r o b l e m. Determine the angle α to the horizontal at which the load on the inclined plane remains in equilibrium if the coefficient of friction is f_0.



S o l u t i o n. The problem requires that all possible positions for the equilibrium of the load be determined. For this, let us first establish the position of impending motion at which $\alpha = \alpha_l$. In that position acting on the load are its weight $\boldsymbol{P}$, the normal reaction $\boldsymbol{N}$ and the limiting friction $\boldsymbol{F_l}$.

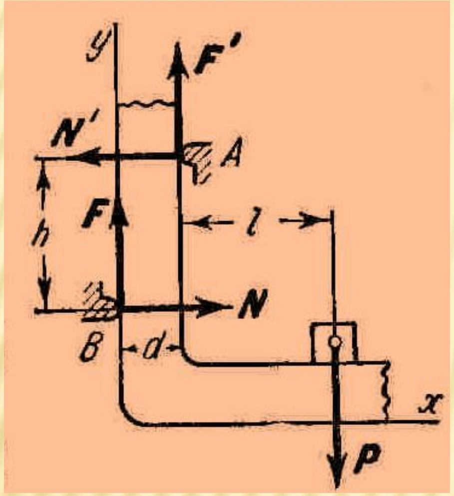
Constructing a closed triangle with these forces, we find that $F_l = N \tan \alpha_l$. But, on the other hand, $F_l = f_0 N$. Consequently, $\tan \alpha_l = f_0$.

In this equation α_l decreases as f_0 decreases. We conclude, therefore, that equilibrium is also possible at $\alpha < \alpha_l$. Finally, all the values of α at which the load remains in equilibrium are determined by the inequality

$$\tan \alpha \leq f_0.$$

If there is no friction ($f_0 = 0$), equilibrium is possible only at $\alpha = 0$.

Problem. A bent bar whose members are at right angles is constrained at A and B as shown in Fig. The vertical distance between A and B is h. Neglecting the weight of the bar, determine the thickness d at which the bar with a load lying on its horizontal member will remain in equilibrium regardless of the location of the load. The coefficient of static friction of the bar on the constraints is f_0.



S o l u t i o n. Let us denote the weight of the load by P and its distance from the vertical member of the bar by l. Now consider the position of impending slip of the bar, when $d = d_l$. In this position acting on it are force $\boldsymbol{P}$, $\boldsymbol{N}$, $\boldsymbol{F}$, $\boldsymbol{N}'$, and $\boldsymbol{F}'$, where $\boldsymbol{F}$ and $\boldsymbol{F}'$ are the forces of limiting friction.

Writing the equilibrium equations, taking the moments about A, we obtain:

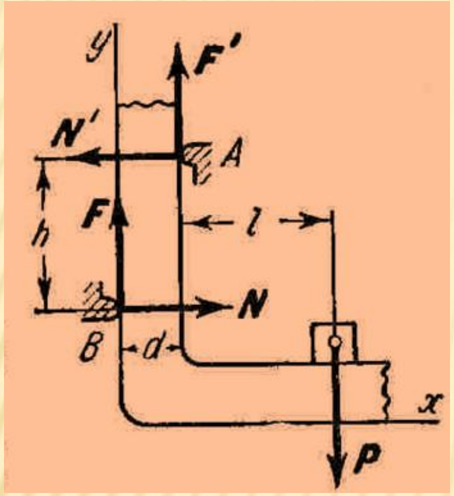
$$N - N' = 0, \qquad F + F' - P = 0, \qquad Nh - Fd_l - Pl = 0,$$

where $F = f_0 N$ and $F' = f_0 N'$. From the first two equations we find:

$$N = N', \qquad P = 2f_0 N.$$

Substituting these values in the third equation and eliminating N, we have:

$h - f_0 d_l - 2 f_0 l = 0,$ whence $d_l = \frac{h}{f_0} - 2l.$



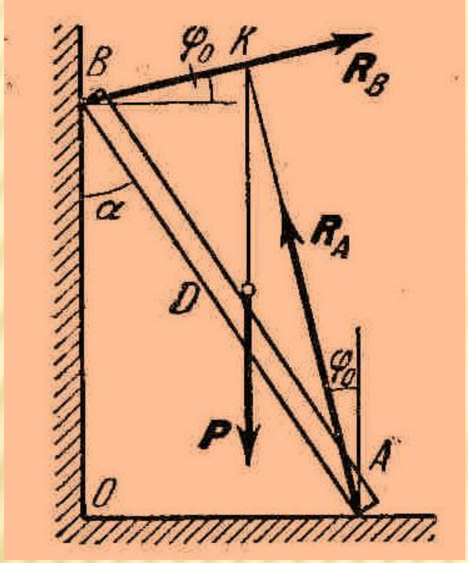
Let us analyze this result.

If in this equation we reduce f_0 the right-hand side will tend to infinity. Hence, equilibrium is possible at any value of $d > d_l$. The maximum value of d_l is at $l = 0$.

Thus, the bar will remain in equilibrium wherever the load is placed (at $l>0$) if the inequality $d \geq \frac{h}{f_0}$ is satisfied.

The less the friction the grater must d be, If there is no friction ($f_0=0$) equilibrium is obviously impossible, as $d = \infty$.

P r o b l e m. Neglecting the weight of the ladder *AB* in Fig., determine the values of angle α at which a man can climb to the top of the ladder at *B* if the angle of friction for the contacts at the floor and the wall is φ_0.



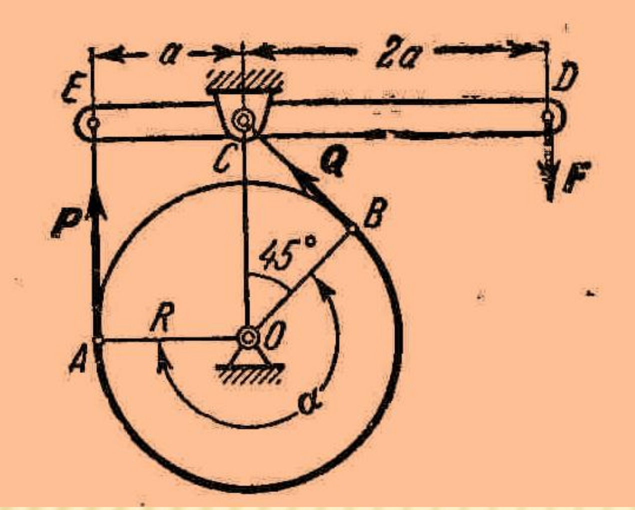
S o l u t i o n. Let us examine the position of impending slip of the ladder by graphical method. For impending motion the forces acting on the ladder are the reactions of the floor and wall $\boldsymbol{R_A}$ and $\boldsymbol{R_B}$ which are inclined at the angle of friction φ_0 to the normals of the surfaces. The action lines of the reaction intersect at *K*.

Thus, for the system to be in equilibrium the third force ***P*** acting on the ladder must also pass through *K*.

Hence, in the position shown the man cannot climb higher than *D*. For him to reach *B* the action lines of $\boldsymbol{R_A}$ and $\boldsymbol{R_B}$ must intersect somewhere along *BO*, which is possible only if force $\boldsymbol{R_A}$ is directed along *AB*, i.e., when $\alpha \leq \varphi_0$.

Thus a man can climb to the top of a ladder only if its angle with the wall does not exceed the angle of friction with the floor. The friction on the wall is irrelevant, i.e., the wall may be smooth.

P r o b l e m. A force $\boldsymbol{F}$ is applied to the lever *DE* of the band-brake. Determine the frictional torque M_T exerted on the drum of radius *R*, if $CD = 2CE$ and the coefficient of friction of the band on the drum is $f_0 = 0.5$.



S o l u t i o n. Acting on the drum and band *AB* wrapped around it is a force $\boldsymbol{P}$ (evidently $P=2F$) applied at *A* and a force $\boldsymbol{Q}$ applied at *B*.

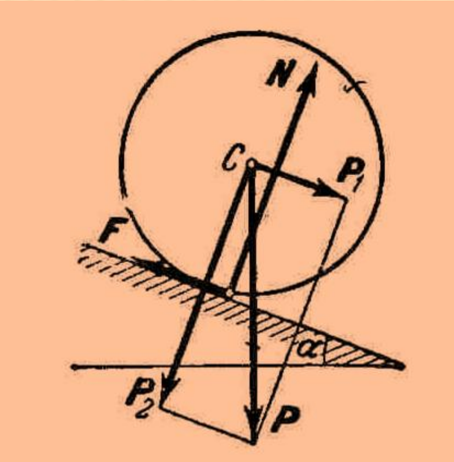
We also have $f_0 = 0.5$ and $\alpha = \frac{5}{4}\pi = 3.93$ radians.

Hence, $Q = 2Fe^{-\frac{5}{8}\pi} \approx 0.28\,F$.

The required torque is $M_T = (P - Q)R = 1.72FR$.

The less the value of *Q*, i.e. the grater the coefficient of friction f_0 and the angle α, the greater the torque.

Problem. Determine the values of angle α at which a cylinder of radius R will remain at rest on an inclined plane if the coefficient of rolling friction is k.



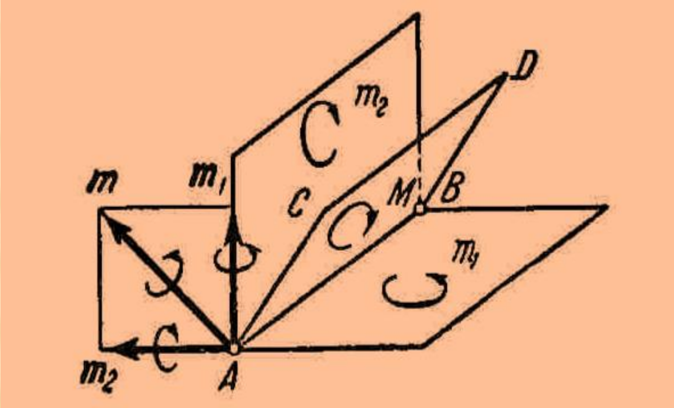
S o l u t i o n. Consider the position of impending motion, when $\alpha = \alpha_l$.

Resolving force $\boldsymbol{P}$ into rectangular components $\boldsymbol{P_1}$ and $\boldsymbol{P_2}$, we find that the moving force $Q_l = P_1 = P\sin\alpha_l$, and the normal reaction $N = P_2 = P\cos\alpha_l$.

We have: $P\sin\alpha_l = \frac{k}{R}P\cos\alpha_l$, or $\tan\alpha_l = \frac{k}{R}$.

If k tend to zero the value of α_l also tends to zero. We conclude from this that equilibrium is maintained at any angle $\alpha < \alpha_l$.

P r o b l e m. Acting on a rigid body are two couples in mutually perpendicular planes. The moment of each is 30 N-m. Determine the resultant couple.

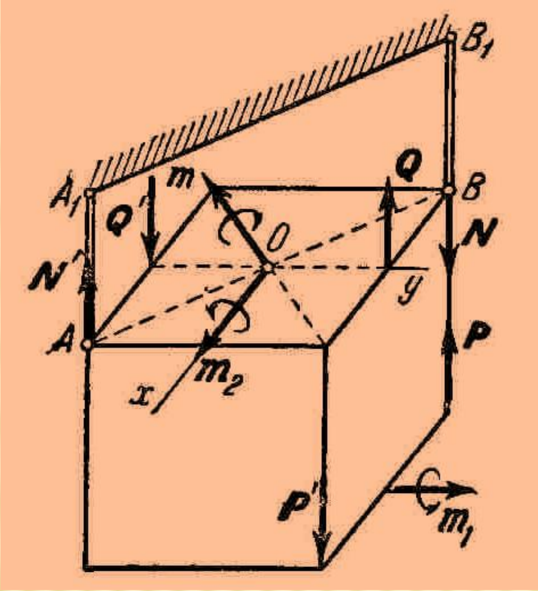


S o l u t i o n. Denote the moments of the two couples by vectors $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ applied at an arbitrary point *A*; the moment of the resultant couples is denoted by vector $\boldsymbol{m}$.

The resultant couple is located in plane *ABCD* normal to $\boldsymbol{m}$ and the magnitude of the resultant moment is $30\sqrt{2}$ N-m.

If the sense of rotation of one of given couples is reversed, the resultant couple will occupy a plane normal to *ABCD*.

P r o b l e m. The cube hangs from two vertical rods AA_1 and BB_1 so that its diagonal AB is horizontal. Applied to the cube are couples ($\boldsymbol{P}$, $\boldsymbol{P'}$) and ($\boldsymbol{Q}$,$\boldsymbol{Q'}$). Neglecting the weight of the cube, determine the relation between forces P and Q at which it will be in equilibrium and the reactions of the rods.



S o l u t i o n. The system of couples ($\boldsymbol{P}$, $\boldsymbol{P'}$) and ($\boldsymbol{Q}$, $\boldsymbol{Q'}$) is equivalent to a couple and can be balanced only by a couple. Hence, the required reactions $\boldsymbol{N}$ and $\boldsymbol{N'}$ must form a couple.

Let us denote its moment $\boldsymbol{m}$ normal to diagonal AB as shown in the diagram. In scalar magnitude $m = Na\sqrt{2}$, where a is the length of the edge of the cube.

Denote the moments of the given couples by the symbols $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$: their scalar magnitudes are $m_1=Pa$ and $m_2=Qa$.

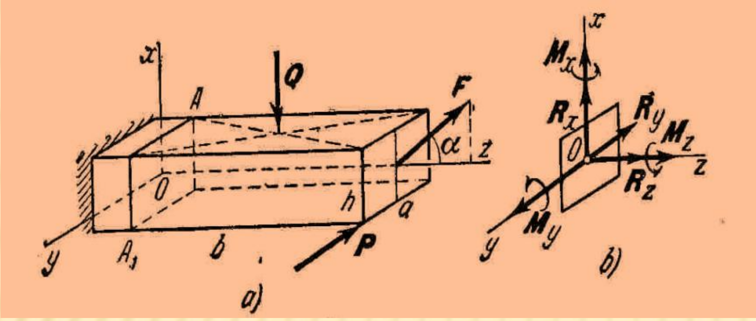
Write the equilibrium equations:

$$\sum m_{kx} = m_2 - m\cos 45° = 0, \qquad \sum m_{ky} = m_1 - m\cos 45° = 0.$$ Whence:

$m_1 = m_2$, i.e., $Q=P$. $m = \dfrac{m_1}{\cos 45°} = m_1\sqrt{2} = Pa\sqrt{2}$. But $m = Na\sqrt{2}$, hence $N=P$.

Thus, equilibrium is possible when $Q=P$. The reactions of the rods are equal to P in magnitude and are directed as shown.

P r o b l e m. Determine the stresses in section AA_1 of a beam. Force $\boldsymbol{Q}$ goes through the centre of the right-hand portion of the beam; force F lies in the plane Oxz; force $\boldsymbol{P}$ is parallel to the y axis.



S o l u t i o n. For this reduce all the forces to the centre O of the section, and place the origin of the coordinate system there.

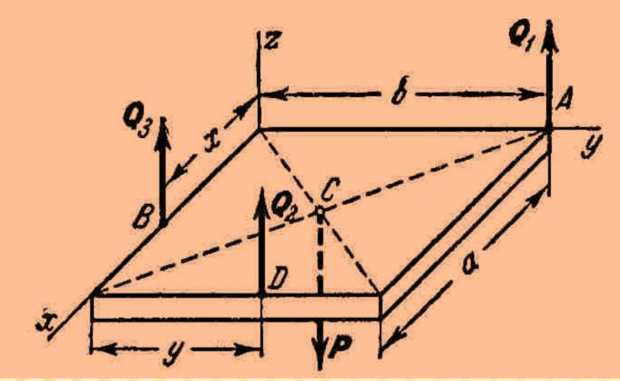
To determine the principal vector and principal moment of the system, we have:

$$R_x = F\sin\alpha - Q, \qquad R_y = -P, \qquad R_z = F\cos\alpha;$$

$$M_x = bP, \qquad M_y = bF\sin\alpha - \frac{b}{2}Q, \qquad M_z = \frac{h}{2}P.$$

Thus, acting on the section AA_1 are two lateral forces R_x and R_y, an axial tension R_z, and three couples of moments M_x, M_y, and M_z (Fig. b): the first two tend to bend the beam about axes Ox and Oy and the last tends to twist it about axis Oz.

P r o b l e m. Three workers lift an homogeneous rectangular plate whose dimensions are a by b . If one worker is at A, determine the points B and D where the other workers should stand so that they would all exert the same force.



S o l u t i o n. The plate is a free body acted upon by four parallel forces $\boldsymbol{Q_1}$, $\boldsymbol{Q_2}$, $\boldsymbol{Q_3}$ and $\boldsymbol{P}$, where $\boldsymbol{P}$ is the weight of the plate.

Assuming that the plate is horizontal, we obtain from the equilibrium conditions:

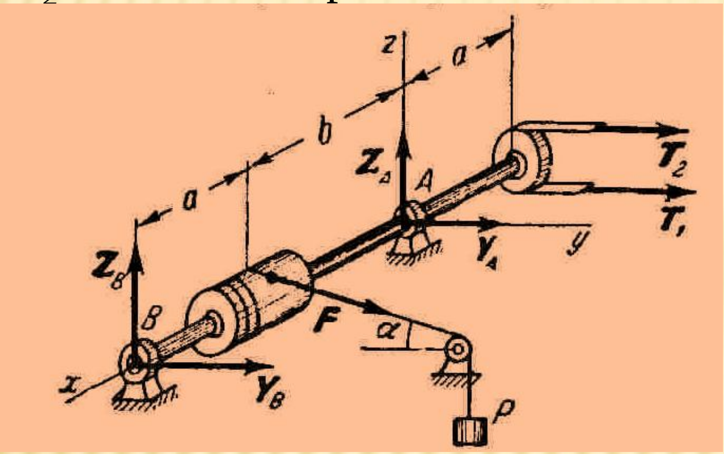
$$Q_1 b - Q_2 y - P\frac{b}{2} = 0, \quad -Q_2 a - Q_3 x + P\frac{a}{2} = 0, \qquad Q_1 + Q_2 + Q_3 = P.$$

According to the conditions of the problem, $Q_1 = Q_2 = Q_3 = Q$, hence, from the last equation, $P = 3Q$.

Substituting this expression in the first two equations and eliminating Q, we have:

$$b + y = \frac{3}{2}b, \qquad a + x = \frac{3}{2}a, \qquad x = \frac{a}{2}, \qquad y = \frac{b}{2}.$$

P r o b l e m. A horizontal shaft supported in bearings *A* and *B* as shown has attached at right angles to it a pulley of radius r_1 = 20 *cm* and a drum of radius r_2=15 *cm*. The shaft is driven by a belt passing over the pulley; attached to a cable wound on the drum is a load of weight *P* = 1800 *N* which is lifted with uniform motion when the shaft turns. Neglecting the weight of the construction, determine the reactions of the bearings and the tension T_1 in the driving portion of the belt, if it is known that, it is double the tension T_2 in the driven portion and if *a*=40 *cm*, *b*=60 *cm*, and $\alpha = 30°$.



S o l u t i o n. As the shaft rotates uniformly, the forces acting on it are in equilibrium and the equations of equilibrium can be applied.

From the equilibrium equations, and noting that *F* = *P*, we obtain:

$$P\cos\alpha + T_1 + T_2 + Y_A + Y_B = 0,$$

$$-P\sin\alpha + Z_A + Z_B = 0, \quad -r_2P + r_1T_1 - r_1T_2 = 0, \quad bP\sin\alpha - (a+b)Z_B = 0,$$

$$bP\cos\alpha - aT_1 - aT_2 + (a+b)Y_B = 0.$$

Remembering that $T_1 = 2T_2$, we find immediately from the third and fourth equations that $T_2 = \dfrac{r_2P}{r_1} = 1350\ N, \quad Z_B = \dfrac{bP}{a+b}\sin\alpha = 540\ N.$

From the fifth equation we obtain $Y_B = \frac{3aT_2 - bP\cos\alpha}{a+b} \approx 690\ N$.

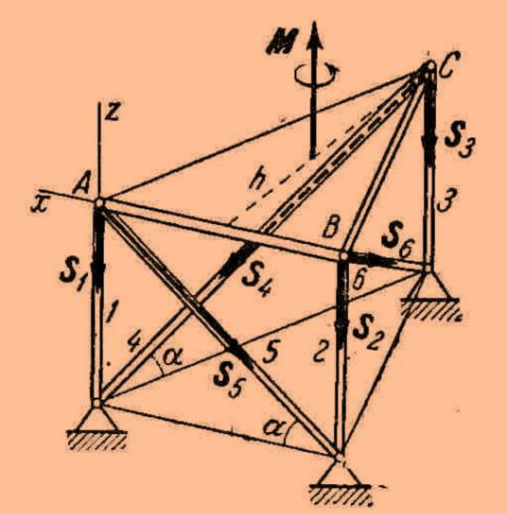
Substituting these values in other equations we find:

$$Y_A = -P\cos\alpha - 3T_2 - Y_B \approx -6300\ N, \qquad Z_A = P\sin\alpha - Z_B = 360\ N,$$

and finally,

$$T_1 = 2700\ N, Y_A \approx -6300\ N, Z_A = 360\ N, Y_B \approx 690\ N, Z_B = 540\ N.$$

Problem. An equilateral triangular plate with sides of length a is supported in a horizontal plane by six bars as shown in fig. Each inclined bar makes an angle of $\alpha = 30°$ with the horizontal. Acting on the plate is a couple of moment *M*. Neglecting the weight of the plate, determine the stresses produced in the bars.



S o l u t i o n. Regarding the plate as a free body, draw, as shown in the figure, the vector of moment ***M*** of the couple and the reactions of the bars $\boldsymbol{S_1}, \boldsymbol{S_2}, \ldots, \boldsymbol{S_6}$.

Writing the equations of the moment with respect to that *x*-axis, we obtain, as $M_z = M$,

$$(S_6 \cos\alpha)h + M = 0,$$

where $h = \frac{a\sqrt{3}}{2}$ is the altitude of the triangle.

From this we find: $S_6 = -\frac{2\sqrt{3}}{3}\frac{M}{a\cos\alpha}$.

Writing the equations of the moments with respect to the axes along bars 2 and 3, we obtain similar results for forces S_4 and S_5.

Now write the equations of the moments about axis x, which is directed along side *BA* of the triangle. Taking into account that $M_x = 0$, we obtain $S_3 h + (S_4 \sin\alpha)h = 0$, whence, as $S_4 = S_6$, we find

$$S_3 = -S_4 \sin\alpha = \frac{2\sqrt{3}}{3}\frac{M}{a}\tan\alpha\,.$$

Writing the moment equations with respect to axes *AC* and *CB*, we obtain similar results for S_1 and S_2.

Finally, for $\alpha = 30°$, we have:

$$S_1 = S_2 = S_3 = \frac{2}{3}\frac{M}{a};\quad S_4 = S_5 = S_6 = -\frac{4}{3}\frac{M}{a}.$$

The answer shows that the given couple creates tensions in the vertical bars and compressions in the inclined ones.